Hyperchaos in the Post-Breakdown Regime of p-Germanium

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p-Ge electrically driven into the post-breakdown regime at liquid-He temperatures produces voltage oscillations which can be attributed to the formation of a chaotic attractor. Under variation of an applied magnetic field, a change in this attractor takes place which apparently reflects an increase in attractor dimensionality. A sequence of phase plots is presented which is interpreted as a transition from ordinary chaos to hyperchaos.

Recently, the chaotic behavior of nonlinear dynamical systems has attracted considerable attention [1]. A chaotic hierarchy characterized by different chaotic regimes has been proposed by Rössler [2]. According to this theory, the maximum complexity of dynamical behavior possible in nonlinear systems depends only on the number of state variables involved. If the nonlinear dynamical system is dissipative, it can be described by an o.d.e. of the form

$$\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{x}(t) = \boldsymbol{F}(\boldsymbol{x}(t)) \quad (\boldsymbol{x}(t) = \text{state variable}). \quad (1)$$

$\boldsymbol{F}$ is the vector field of the variable $\boldsymbol{x}(t)$. The dimension n of the system can vary from one to infinity. Each dissipative nonlinear system must possess at least one attractor with a dimension less than n. Consequently, two-variable systems can possess only a point attractor or a one-dimensional attractor (limit cycle). Three-variable systems display two more complicated types of behavior, namely a toroidal attractor and a chaotic one. In four and more variables, analogously we can have the hypertoroidal and the hyperchaotic attractor. In the n-dimensional case, all lower-dimensional attractors as well as their combination (mixed cases) are possible. Using the concept of higher-dimensional attractors, specific chaotic behavior can be described. Examples in the three-dimensional case are the Lorenz and the Rössler attractor. Both demonstrate the two main features of chaotic behavior: (i) sensitive dependence on initial conditions [3], (ii) boundedness of the expanding chaotic motion by a backfolding process.

In this paper we discuss these concepts in terms of experimental observations on the post-breakdown behavior of p-germanium at 4.2 K. Experimental studies of different scenarios (routes from stable states to chaos) in current-carrying semiconductors have been reported recently by several groups [4–7]. However, so far the intrinsic dynamical behavior of the chaotic state has not been investigated. In the following, we extend our previous observations of spontaneous oscillations and different routes to chaos in these systems [7]. We find a transition from an ordinary chaotic state to a hyperchaotic state, with a sequence of strange attractors of differing dimensionality. The increase of the dimension of the attractor is perhaps analogous to the emergence of new actively participating degrees of freedom [8] as in a quasi Ruelle-Takens-Newhouse scenario observed previously [7].

Our experimental set-up was similar to that described in [7]: The same single-crystalline p-doped Ge sample was used. The sample dimensions and the arrangement of the ohmic contacts are indicated in Figure 1. An electric field was applied to the outer contacts (voltage V_0). A magnetic field perpendicular to the sample surface could also be applied using a superconducting coil surrounding the Cu shield employed for protection against external irradiation. The inner contacts were used for detecting the voltages V_1 and V_2 (see Figure 1).
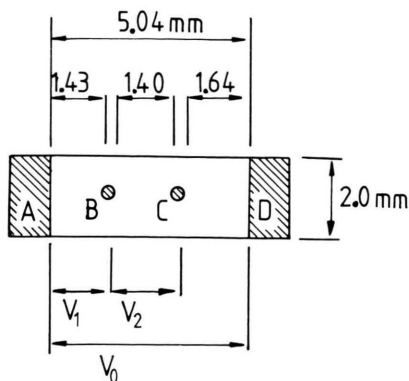
Fig. 1. Sample geometry (thickness 0.25 mm). The hatched areas A–D indicate the ohmic contacts.

During the experiments the sample was in direct contact with the liquid-He bath at 4.2 K. When an electric field of about 5 V/cm was applied, impurity impact ionization breakdown took place, resulting in the generation of a highly nonlinear current-voltage characteristic. In the post-breakdown regime investigated, the current was typically a few mA. Spontaneous current oscillations (of amplitude I_1) were found to be superimposed upon the steady dc current. I_1 was typically a few μA.

In Figure 2 (top), periodic oscillations of the voltages V_1 and V_2 are shown. A phase portrait V_1 versus V_2 is presented in the middle and shows that both voltages are oscillating in the form of a limit cycle, i.e., there exists rigid coupling between V_1 and V_2. At the bottom we show the power spectrum of $V_2(t)$. The first peak is 35 dB above the noise level.

Variations in the electric or the magnetic field resulted in different oscillatory or chaotic states. A sequence of different chaotic regimes is shown in Figure 3. Part (a) represents a chaotic state obtained for a bias voltage $V_0 = 2.145$ V and a magnetic field of 31.5 G. In all four pictures, the spectral noise power has clearly increased by about 20 dB when compared to the situation in Figure 2. In part (a), middle, the phase portrait is suggestive of a strange attractor: notice the apparent divergence of neighboring trajectories and the backfolding. The dimension of this attractor must be larger than two. The attractor can be visualized as a curled band partly folded over, imbedded in three-dimensional space. This impression was especially striking when the electric field was slightly varied in the 0.1 per cent
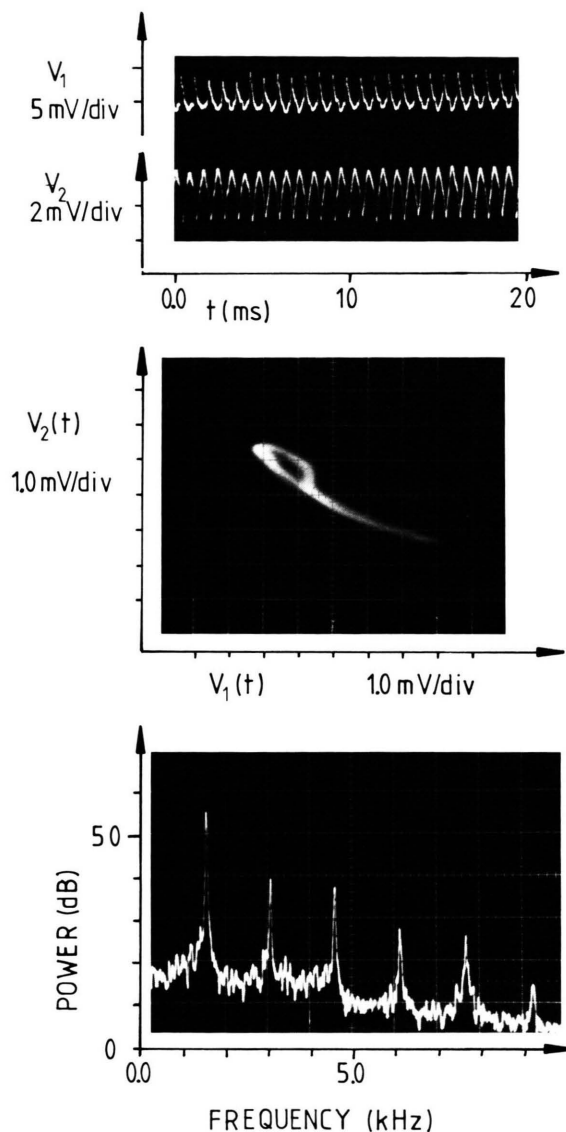

Fig. 2. Limit-cycle oscillations of the voltages $V_1(t)$ and $V_2(t)$ at the bias voltage $V_0 = 2.184$ V; top: V_1 and V_2 plotted against time; middle: phase portrait V_1 vs. V_2; bottom: power spectrum of $V_2(t)$.

range – resulting in a different projection of the same object. Increasing the magnetic field up to 46.5 G led to a structural change of the attractor. This is illustrated in Figures 3b–d. The "curled-band structure" gradually changed into a "tangle". This tangle must have a dimension higher than that of the curled band. Apparently it represents a higher state of chaos – "hyperchaos". The trajec-
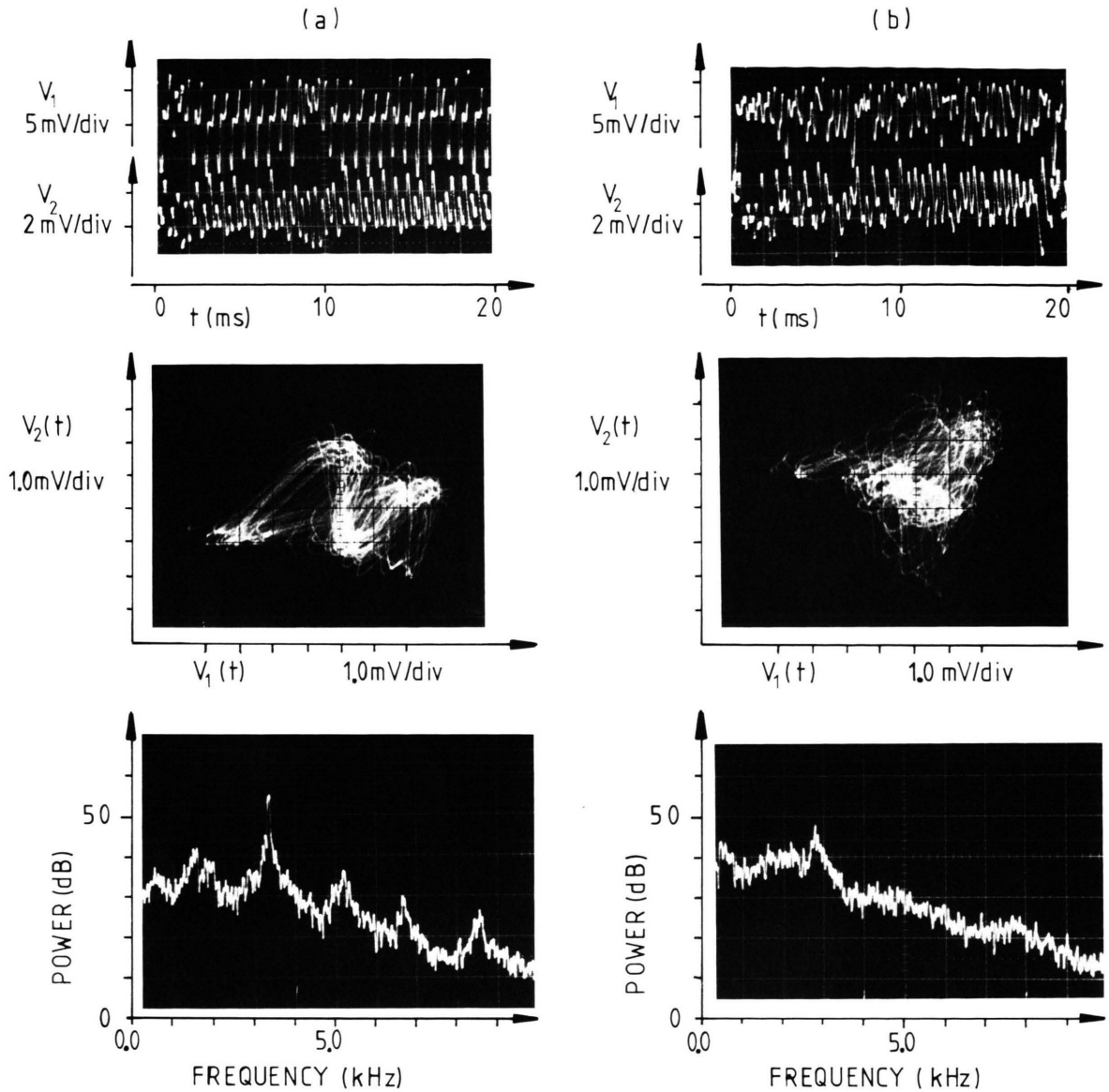
Fig. 3. Different chaotic regimes at the applied magnetic field $B = 31.5$ G (a), $B = 34.0$ G (b), $B = 40.0$ G (c), $B = 46.5$ G (d); the bias voltage was always kept at $V_0 = 2.145$ V; top: V_1 and V_2 plotted against time; middle: phase portraits V_1 vs. V_2; bottom: power spectra of $V_2(t)$.

tories occupy the interior of a nearly spherical portion of the projected phase space. This picture did not change under small variations of the control parameters.

As to our interpretation, the class of systems comprising that we have studied is known to be described by a dissipative hydrodynamics-type

equation [9], cf. [7]. Therefore, it is not too surprising that turbulence-like phenomena can be found experimentally. As in hydrodynamics, not only "beginning turbulence" – that is, ordinary chaos [10] – can be found, but also more fully developed states. The picture of Fig. 3b, middle part, indeed closely resembles a phase portrait recently obtained
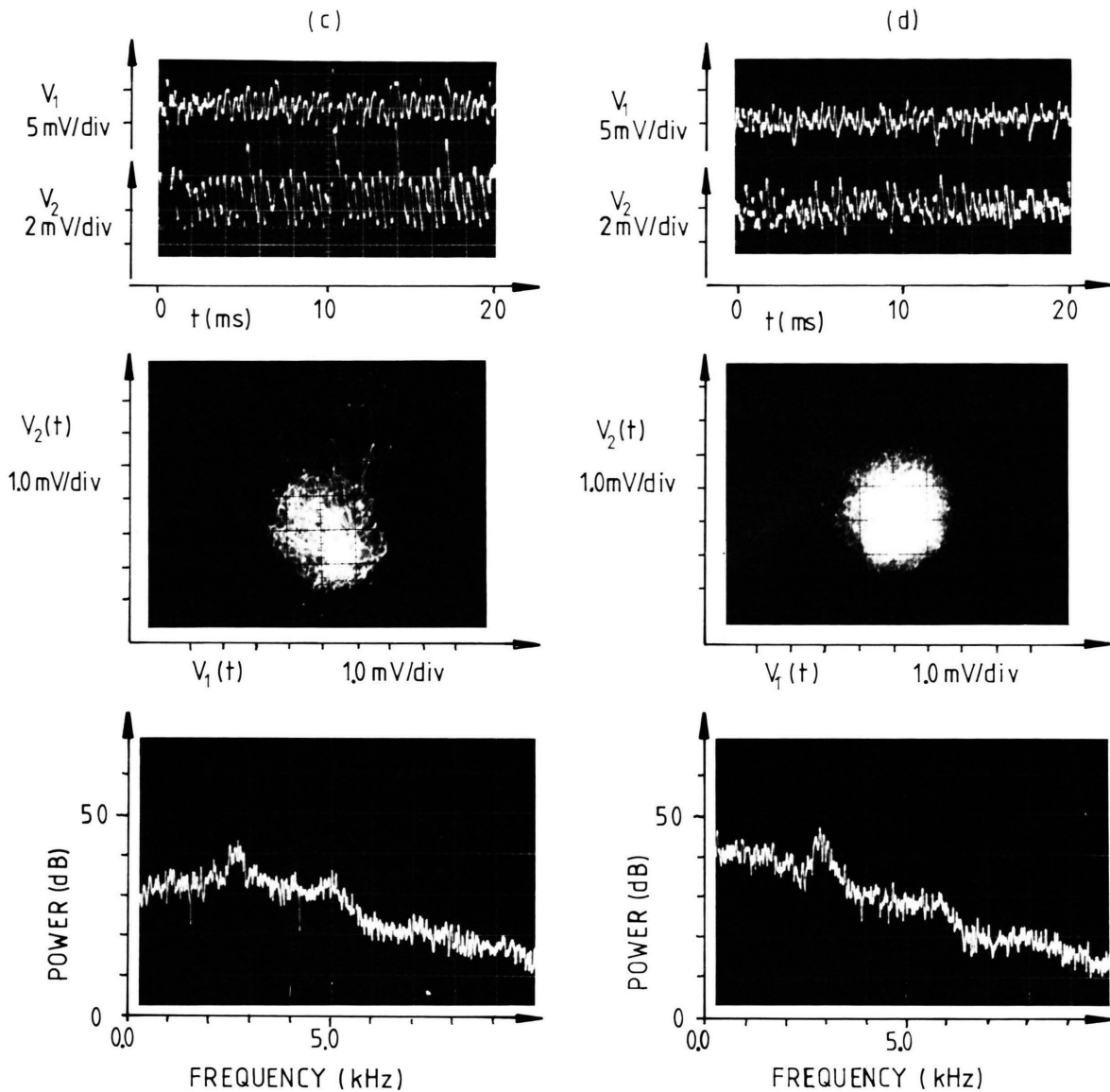
Fig. 3. c, d

in a hydrodynamic system at a parameter value where the previously existing "ordinary chaos" had already disappeared (see Fig. 1a, third part, of [11]). Interestingly, a picture like the spherical tangle (Fig. 3d, above) has apparently not yet been observed in hydrodynamic experiments. We expect that a fairly large number of positive Lyapunov characteristic exponents [12] can eventually be demonstrated in this regime.

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